CHE323/CHE384
Chemical Processes for Micro- and Nanofabrication

# Formulas
# Lectures 1-19

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## Doping, Conductivity, Resistance

- Doping charge balance: $N_D^+ + p = N_A^- + n$
- Mass action equation: $np = n_i^2$

$N_A$ = acceptor concentration; $N_D^+$ = ionized donor concentration
$N_D$ = donor concentration; $n$ = mobile electron concentration
$N_A^-$ = ionized acceptor conc.; $p$ = mobile hole concentration

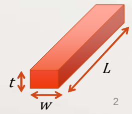
- Resistivity and conductivity:

$$\frac{1}{\rho} = \sigma = q(n\mu_n + p\mu_p)$$

*Resistance* $R = \rho\frac{L}{A}$, $A = wt$

*Sheet Resistance* $R_s = \rho/t$

$q$ = charge on electron = 1.6 X $10^{-19}$ C
$\mu_n$ = electron mobility = 1500 $cm^2/Vs$ for Si at 300K
$\mu_p$ = hole mobility = 450 $cm^2/Vs$ for Si at 300K
$n_i$ = 1.5 X $10^{10}$ $cm^{-3}$ for Si at 300K





## P-N Junction

- Built-in voltage ($V_0$) and the depletion width (W)

$$V_0 = \frac{kT}{q} ln\left(\frac{N_A N_D}{n_i^2}\right) \qquad W = \sqrt{\frac{2\varepsilon_{Si}(V_0 - V)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$


$$\varepsilon_{Si} = 11.7\varepsilon_0, \quad \varepsilon_0 = 8.8542\times10^{-12}\ C/V\,m \qquad \text{At } T = 300\text{ K}, \quad \frac{kT}{q} = 25\,mV$$

- Diode Equation: current $I_{diode} = I_0\left(e^{qV/kT} - 1\right)$

- Capacitance: $C_{p-n\,junction} = \frac{\varepsilon A}{W} = A\sqrt{\frac{q\varepsilon_{Si}}{2(V_0 - V)}\left(\frac{N_D N_A}{N_D + N_A}\right)}$



## Deal-Grove Oxidation Model

$$t_{ox}^2 + At_{ox} = B(t + \tau) \qquad \tau = \frac{t_o^2 + At_o}{B}$$

TABLE 4.1 / OXIDATION COEFFICIENTS FOR SILICON (111) wafers

| Temperature (°C) | Dry A (μm) | Dry B (μm²/hr) | Dry τ (hr) | Wet (640 torr) A (μm) | Wet (640 torr) B (μm²/hr) |
|---|---|---|---|---|---|
| 800 | 0.370 | 0.0011 | 9 | — | — |
| 920 | 0.235 | 0.0049 | 1.4 | 0.50 | 0.203 |
| 1000 | 0.165 | 0.0117 | 0.37 | 0.226 | 0.287 |
| 1100 | 0.090 | 0.027 | 0.076 | 0.11 | 0.510 |
| 1200 | 0.040 | 0.045 | 0.027 | 0.05 | 0.720 |

The τ parameter is used to compensate for the rapid growth regime for thin oxides *(after Deal and Grove)*.

For (100) wafers, multiply A by 1.68.



## Deal-Grove Temperature Dependence

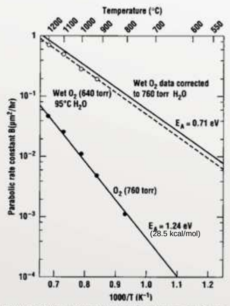



Figure 4.2 Arrhenius plot of the *B* oxidation coefficient. The wet parameters depend on the $H_2O$ concentration and therefore on the gas flows and pyrolysis conditions *(after Deal and Grove)*. (111) wafers

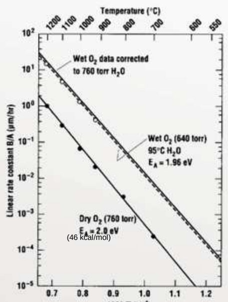



Figure 4.3 Arrhenius plot of the ratio *(B/A)* of the oxidation parameters *(after Deal and Grove)*. (111) wafers



## Diffusion Review

- Fick's 2nd law of Diffusion (in 1-D):

$$\frac{\partial C(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial C(x,t)}{\partial x}\right)$$

- In 3-D: $\frac{\partial C}{\partial t} = \nabla(D\nabla C)$, $\quad \nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$
- Analytical solutions are for $D$ = constant and certain special boundary conditions
- Electric field enhancement: $\eta \approx \frac{C(z)}{\sqrt{C^2(z) + 4n_i^2}}$

$$D_{enhanced} = D(1 + \eta)$$

## Diffusion Case 1: Constant Source

- Initial and boundary conditions
  - $C(0,t) = C_s$ (concentration at top is constant)
  - $C(z,0) = 0$ for $z > 0$ (initial condition)
  - $C(\infty,t) = 0$
- Solution: $C(z,t) = C_s erfc\left(\frac{z}{2\sqrt{Dt}}\right), \quad t > 0$

$\sqrt{Dt}$ = diffusion length (average distance a dopant moves)

$$Q_T(t) = \int_0^\infty C(z,t)dz) = \frac{2}{\sqrt{\pi}} C_s \sqrt{Dt}$$

## Diffusion Case 2: Limited Source

(drive-in diffusion)

- Initial and boundary conditions
  - $C(z,0) = 0, z > 0$
  - $dC(0,t)/dt = 0$ (no flux at top)
  - $C(\infty,t) = 0$
  - Constant dose: $\int_0^\infty C(z,t)dz = Q_T = constant$
- Solution:

$$C(z,t) = \frac{Q_T}{\sqrt{\pi Dt}} e^{-z^2/4Dt}, \qquad t > 0$$

## Case 3: Buried Gaussian Source

- Initial and boundary conditions
  - Gaussian: $C(z,0) = \frac{Q_T}{\sqrt{2\pi\sigma_o{}^2}} e^{-(z-\mu)^2/2\sigma_o{}^2}, \qquad z \geq 0, \mu \gg \sigma_o$
  - $dC(0,t)/dt = 0$ (no flux at top)
  - $C(\infty,t) = 0$
- Solution: $\sigma^2 = \sigma_o{}^2 + 2Dt \qquad (\sqrt{2Dt}$ = diffusion length)

$$C(z,t) = \frac{Q_T}{\sqrt{2\pi\sigma^2}} e^{-(z-\mu)^2/2\sigma^2}, \qquad z \geq 0, t \geq 0$$

## Gaussian Ion Implantation Model

- Gaussian model for the distribution of dopants
  - Mean = $R_p$ = projected range
  - Standard deviation = $\Delta R_p$ = straggle
  - Dose = $\phi$ (# dopants/cm$^2$)

$$N(x) = \frac{\phi}{\sqrt{2\pi}\Delta R_p} e^{-(x-R_p)^2/2\Delta R_p{}^2}$$

- Lateral scattering
  - For As, Sb: $\Delta R_\perp \approx \Delta R_p$
  - For P: $\Delta R_\perp \approx 1.2\Delta R_p$
  - For B: $\Delta R_\perp \approx 2\Delta R_p$

## Ion Implantation Model Parameters

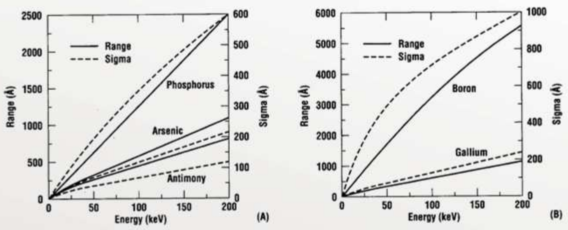



Figure 5.9 Projected range (solid lines and left axis) and standard deviation (dashed lines and right axis) for (A) n-type, (B) p-type, and (C) other species into a silicon substrate; (D) n-type and (E) p-type dopants into a GaAs substrate; and several implants into (F) $SiO_2$ and (G) AZ111 photoresist (*data from Gibbons* et al.).

$$\Delta R_p \approx \frac{2}{3} R_p \left(\frac{\sqrt{M_i M_t}}{M_i + M_t}\right) \qquad i = ion,\ t = target \quad M = atomic\ mass$$

## Thermal Transfer Mechanisms

- Radiative: Stefan-Boltzmann equation

$$Heat\ Flow = \dot{q} = \varepsilon\sigma T^4$$

$\varepsilon$ = emissivity of emitting body ($\varepsilon$ = 1 for black body)
$\sigma$ = Stefan-Boltzmann Constant = 5.6697 x $10^{-8}$ W/m$^2$-K$^4$

- Conduction: $\dot{q} = k\nabla T$
- Convection: $\dot{q} = h(T - T_\infty)$

$$\varepsilon(\lambda) = 1 - R(\lambda) - T(\lambda)$$

$$\varepsilon_{Si} \approx 0.7, \qquad T_{Si} \approx 0$$