Doping, Conductivity, Resistance

- Doping charge balance: \( N_A^+ + p = N_D^- + n \)
- Mass action equation: \( np = n_i^2 \)
  
  \( N_A \) = acceptor concentration
  
  \( N_D \) = donor concentration
  
  \( n_i \) = mobile electron concentration
  
  \( N_A^+ \) = ionized acceptor conc.
  
  \( p \) = mobile hole concentration

- Resistivity and conductivity:
  
  \[ \rho = \frac{1}{\sigma} = \frac{1}{\sigma_{0}} \left( \frac{\mu_n + \mu_p}{q} \right) \]
  
  \[ 1 / \sigma = \frac{1}{\rho} \]  \( \sigma_{0} \)  \( A = \omega t \)

\( q = \) charge on electron = \( 1.6 \times 10^{-19} \) C

\( \mu_n = \) electron mobility = \( 1500 \) \( \text{cm}^2/\text{Vs} \) for Si at 300K

\( \mu_p = \) hole mobility = \( 450 \) \( \text{cm}^2/\text{Vs} \) for Si at 300K

\( n_i = \) \( 1.5 \times 10^{10} \) \( \text{cm}^{-3} \) for Si at 300K

P-N Junction

- Built-in voltage \( (V_B) \) and the depletion width \( (W) \):

  \[ V_B = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) \]

  \[ W = \sqrt{\frac{2q(V_B - V)}{kT}} \left( \frac{1}{N_d} + \frac{1}{N_D} \right) \]

\( \varepsilon_{p} = 11.7 \varepsilon_{0} \), \( \varepsilon_{p} = 8.8542 \times 10^{-12} \) \( \text{C} / \text{Vm} \)  \( \text{At} T = 300 \text{K} \)

\[ \frac{kT}{q} = 25 \text{mV} \]

- Diode Equation: current \( I_{\text{diode}} = I_0 (e^{qV_B/kT} - 1) \)

- Capacitance: \( C_{\text{P-N junction}} = \frac{\varepsilon A}{W} = \frac{A q e_{p}}{2(V_B - V)} \left( \frac{N_D N_A}{N_D + N_A} \right) \)

Deal-Grove Oxidation Model

\[ t_{ox}^2 + A t_{ox} = B(t + \tau) \]

\[ \tau = \frac{t_{ox}^2 + A t_{ox}}{B} \]

<table>
<thead>
<tr>
<th>TABLE 11.4</th>
<th>OXIDATION COEFFICIENTS FOR SILICON (II) WAFERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A ) (( \text{A} ))</td>
<td>( B ) (( \text{A} ))</td>
</tr>
<tr>
<td>( 800 )</td>
<td>0.370</td>
</tr>
<tr>
<td>( 920 )</td>
<td>0.235</td>
</tr>
<tr>
<td>( 1000 )</td>
<td>0.165</td>
</tr>
<tr>
<td>( 1100 )</td>
<td>0.090</td>
</tr>
<tr>
<td>( 1200 )</td>
<td>0.040</td>
</tr>
</tbody>
</table>

\( A \) (\( \text{A} \)) \( B \) (\( \text{A} \))

\( A \) (\( \text{A} \)) \( B \) (\( \text{A} \))

\( \text{Dry} \) \( \text{Wet (640 Ke)} \)

\( The \ \tau \ parameter \ is \ used \ to \ compensate \ for \ the \ rapid \ growth \ regime \ for \ thin \ oxides \ (after \ Deal \ and \ Grove). \)

For \( 100 \) wafers, multiply \( A \) by \( 1.68 \).

Deal-Grove Temperature Dependence

- Fick’s 2\textsuperscript{nd} law of Diffusion (in 1-D):

  \[ \frac{\partial C(x, t)}{\partial t} = \frac{\partial}{\partial x} \left( D \frac{\partial C(x, t)}{\partial x} \right) \]

- In 3-D:

  \[ \frac{\partial C}{\partial t} = \nabla (D \nabla C), \quad \nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \]

- Analytical solutions are for \( D = \) constant and certain special boundary conditions

- Electric field enhancement:

  \[ \eta = \frac{C(z)}{ \sqrt{C(z)^2 + 4n_i^2} } \]

\[ D_{\text{enhanced}} = D(1 + \eta) \]

\( C(z) \) \( D \) \( \eta \)

\( D \) \( \eta \) \( C(z) \)

\( D \) \( \eta \) \( C(z) \)

\( D \) \( \eta \) \( C(z) \)
Diffusion Case 1: Constant Source

- Initial and boundary conditions
  - \( C(0,t) = C_s \) (concentration at top is constant)
  - \( C(z,0) = 0 \) for \( z > 0 \) (initial condition)
  - \( C(\infty,t) = 0 \)

- Solution:
  \[
  C(z,t) = C_s \text{erf} \left( \frac{z}{2\sqrt{Dt}} \right), \quad t > 0
  \]

\[ \sqrt{Dt} \text{ = diffusion length (average distance a dopant moves)} \]

\[ Q_T(t) = \frac{2}{\sqrt{\pi}} C_s \sqrt{Dt} \]

Diffusion Case 2: Limited Source

- Initial and boundary conditions
  - \( C(z,0) = 0, \ z > 0 \)
  - \( \frac{dC(0,t)}{dt} = 0 \) (no flux at top)
  - \( C(\infty,t) = 0 \)

- Constant dose:
  \[
  C(z,t) = \frac{Q_T}{\sqrt{\pi Dt}} e^{-z^2/4Dt}, \quad t > 0
  \]

Case 3: Buried Gaussian Source

- Initial and boundary conditions
  - Gaussian:\n    \[
    C(0,t) = \frac{Q_T}{2\pi\sigma_0^2} e^{-t^2/2\sigma_0^2}, \quad z \geq 0, \ t > 0
    \]
  - \( \frac{dC(0,t)}{dt} = 0 \) (no flux at top)
  - \( C(\infty,t) = 0 \)

- Solution:
  \[
  C(z,t) = \frac{Q_T}{\sqrt{2\pi\sigma^2}} e^{-t^2/2\sigma^2}, \quad z \geq 0, \ t > 0
  \]

Gaussian Ion Implantation Model

- Gaussian model for the distribution of dopants
  - Mean = \( R_p \) = projected range
  - Standard deviation = \( \Delta R_p \) = straggle
  - Dose = \( \phi \) (# dopants/cm\(^2\))

- Lateral scattering
  - For As, Sb: \( \Delta R_{\perp} = \Delta R_p \)
  - For P: \( \Delta R_{\perp} = 1.2 \Delta R_p \)
  - For B: \( \Delta R_{\perp} = 2 \Delta R_p \)

Ion Implantation Model Parameters

- Thermal Transfer Mechanisms
  - Radiative: Stefan-Boltzmann equation
    \[
    \text{Heat Flow} = \dot{q} = \varepsilon\sigma T^4
    \]
  - \( \varepsilon \) = emissivity of emitting body (\( \varepsilon = 1 \) for black body)
  - \( \sigma \) = Stefan-Boltzmann Constant = 5.6679 \times 10^{-8} \text{ W/m}^2\text{-K}^4

- Conduction: \( \dot{q} = k T \)
- Convection: \( \dot{q} = h(T - T_{\infty}) \)

\[
\varepsilon(\lambda) = 1 - R(\lambda) - T(\lambda)
\]

\( \varepsilon_{Si} \approx 0.7, \quad T_{Si} \approx 0 \)