

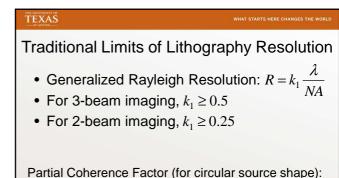
THE UNIVERSITY OF TEXAS		WHAT STA	RTS HERE CHANGES THE WORLD
Fourier Transform Examples	$g(x)$ $rect(x) = \begin{cases} 1, &  x  < 0.5 \\ 0, &  x  > 0.5 \end{cases}$	Graph of g(x)	$\frac{G(f_x)}{\frac{\sin(\pi f_x)}{\pi f_x}}$
$\mathcal{F}\{g(x)\} = G(f_x)$	$step(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$		$\frac{1}{2}\delta(f_x) - \frac{i}{2\pi f_x}$
Delta Function:	Delta function $\delta(x)$		1
$\delta(x) = 0  when  x \neq 0$	$comb(x) = \sum_{j=-\infty}^{\infty} \delta(x-j)$	1111111	$\sum_{j=-\infty}^{\infty} \delta(f_x - j)$
$\int_{0}^{\infty} S(x) dx = 1$	cos(#x)	<b>-</b> ∕∕∳∕\-	$\frac{1}{2}\delta\!\!\left(f_x\!+\!\frac{1}{2}\right)\!+\!\frac{1}{2}\delta\!\!\left(f_x\!-\!\frac{1}{2}\right)$
$\int_{-\infty} \delta(x)  dx = 1$	sin(Æ)	<b>-√</b>   <b>/</b> \	$\frac{i}{2}\delta\!\!\left(f_x+\!\frac{1}{2}\right)\!-\!\frac{i}{2}\delta\!\!\left(f_x-\!\frac{1}{2}\right)$
$\int_{0}^{\infty} f(x)\delta(x-x_{o})dx = f(x_{o})$	Gaussian $e^{-\pi x^2}$	<u></u>	$e^{-\pi f_x^2}$
—∞ © Chris Mack, 2013	$circ(r) = \begin{cases} 1, &  r  < 1 \\ 0, &  r  > 1 \end{cases}$ $r = \sqrt{x^2 + y^2}$		$\rho = \sqrt{f_x^2 + f_y^2}$

— AT AUSTIN —		
Fourier Optics		
• Pupil function, $P(f_x f_y)$		
$P(f_x, f_y) = \begin{cases} 1, & when \sqrt{f_x^2 + f_y^2} \le NA/\lambda \\ 0, & otherwise \end{cases}$		
• Diffraction pattern is $T_m$ , the electric field of the image is $E$ , image intensity is $I$		
$E(x, y) = \mathcal{F}^{-1} \{ PT_m \}$ $I(x, y) =  E(x, y) ^2$		
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TEXAS

## Image Example: equal lines and spaces $T_m(f_x) = \sum_{n=-\infty}^\infty a_n \delta \bigg( f_x - \frac{n}{p} \bigg), \qquad a_n = \frac{\sin(n\pi/2)}{n\pi}$ For case of three diffraction orders going through the lens, $P(f_x)T_m(f_x) = \frac{1}{\pi} \delta \bigg( f_x + \frac{1}{p} \bigg) + \frac{1}{2} \delta (f_x) + \frac{1}{\pi} \delta \bigg( f_x - \frac{1}{p} \bigg)$ $E(x) = F^{-1} \big\{ PT_m \big\} = \frac{1}{\pi} e^{-i2\pi x/p} + \frac{1}{2} + \frac{1}{\pi} e^{i2\pi x/p} = \frac{1}{2} + \frac{2}{\pi} \cos(2\pi x/p)$ With defocus: $\Delta \Phi = 2\pi (OPD)/\lambda = 2\pi \delta (1-\cos\theta)/\lambda$ $E(x) = \frac{1}{2} + \frac{2}{\pi} e^{i\Delta\Phi} \cos(2\pi x/p)$

 $I(x) = \frac{1}{4} + \frac{2}{\pi} \cos(\Delta \Phi) \cos(2\pi x/p) + \frac{2}{\pi^2} \left[ 1 + \cos(4\pi x/p) \right]$ 



 $\sigma = \frac{\text{diameter of illumin ation spot}}{\text{diameter of objective lens entrance pupil}} = \frac{\sin(\theta'_{\text{max}})}{NA_o}$ 

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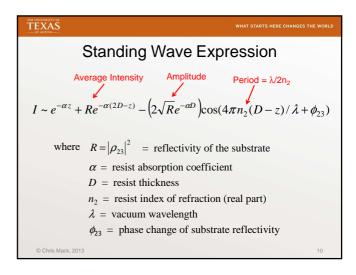
Rayleigh Depth of Focus

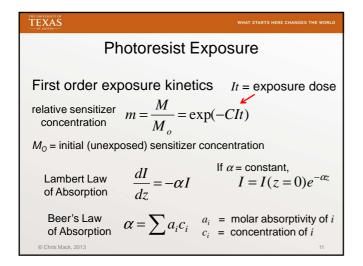
• Three-beam imaging: lines and spaces where only the 0th and ±1st orders are used

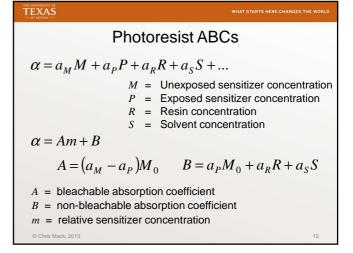
• Feature is at the resolution limit

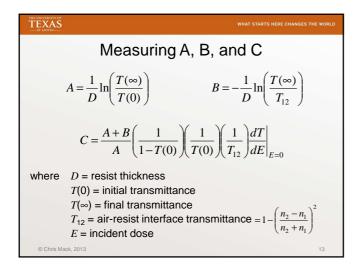
•  $k_2$  is unknown (but it must be < 1)

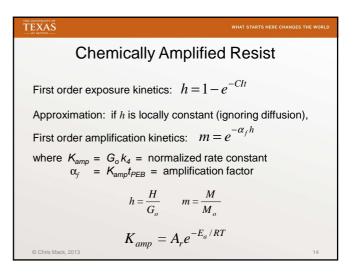
• For low numerical apertures (< 0.5):  $DOF = k_2 \frac{\lambda}{NA^2}$ • For any numerical aperture:  $DOF = \frac{k_2}{2} \frac{\lambda}{n(1-\cos\theta)}$   $\frac{DOF(immersion)}{DOF(dry)} = \frac{1-\sqrt{1-(\lambda/p)^2}}{n-\sqrt{n^2-(\lambda/p)^2}}$ 

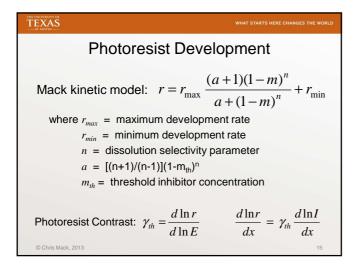


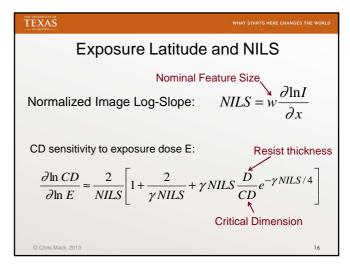












Useful Constants Avogadro Constant 6.02204 X 10<sup>23</sup> mole<sup>-1</sup> 1.38066 X 10<sup>-23</sup> J/K Boltzmann Constant (k)  $8.617 \times 10^{-5} \text{ eV/K}$   $1.3626 \times 10^{-22} \text{ atm-cm}^3/\text{K}$ • Gas Constant (R) 1.987 cal/mole/K • Electric Charge (q) 1.60218 X 10<sup>-19</sup> C Permittivity in vacuum (e<sub>o</sub>) 8.854 X 10<sup>-14</sup> F/cm Thermal voltage at 300 K (kT/q) 0.0259 V • **Pressure**: 1 atm = 1.01325 ×10<sup>5</sup> Pa = 1.01325 bar = 760 torr = 14.696 psi (1 Pa = 1 kg/(m·s²) = 1 N/m²) **Energy**:  $1 J = 1 \text{ kg m}^2/\text{s}^2 = 9.4782 \times 10^{-4} \text{ Btu} = 6.2415 \times 10^{16} \text{ eV} = 0.23901 \text{ cal} = 1 \text{ A V s}$ • Capacitance: 1 F = 1 A s/V = 1 C/V = 1 s/W