

Fourier Transforms. Given $\mathcal{F}\{g(x,y)\} = G(f_x, f_y)$ and $\mathcal{F}\{h(x,y)\} = H(f_x, f_y)$,

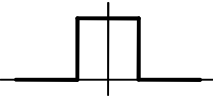
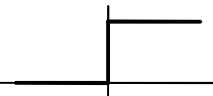
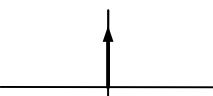


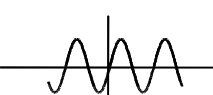
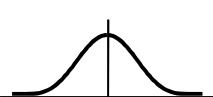
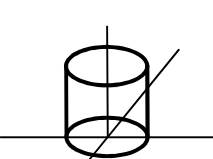
shift theorem $\mathcal{F}\{g(x-a, y-b)\} = G(f_x, f_y)e^{-i2\pi(f_x a + f_y b)}$

similarity theorem $\mathcal{F}\{g(ax, by)\} = \frac{1}{|ab|} G\left(\frac{f_x}{a}, \frac{f_y}{b}\right)$

linearity $\mathcal{F}\{ag(x,y) + bh(x,y)\} = aG(f_x, f_y) + bH(f_x, f_y)$

convolution theorem $\mathcal{F}\left\{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(\xi, \eta)h(x-\xi, y-\eta)d\xi d\eta\right\} = G(f_x, f_y)H(f_x, f_y)$

Table of 1D Fourier transform pairs useful in lithography

$g(x)$	Graph of $g(x)$	$G(f_x)$
$rect(x) = \begin{cases} 1, & x < 0.5 \\ 0, & x > 0.5 \end{cases}$		$\frac{\sin(\pi f_x)}{\pi f_x}$
$step(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$		$\frac{1}{2}\delta(f_x) - \frac{i}{2\pi f_x}$
Delta function $\delta(x)$		1
$comb(x) = \sum_{j=-\infty}^{\infty} \delta(x-j)$		$\sum_{j=-\infty}^{\infty} \delta(f_x - j)$
$\cos(\pi x)$		$\frac{1}{2}\delta\left(f_x + \frac{1}{2}\right) + \frac{1}{2}\delta\left(f_x - \frac{1}{2}\right)$
$\sin(\pi x)$		$\frac{i}{2}\delta\left(f_x + \frac{1}{2}\right) - \frac{i}{2}\delta\left(f_x - \frac{1}{2}\right)$
Gaussian $e^{-\pi x^2}$		$e^{-\pi f_x^2}$
$circ(r) = \begin{cases} 1, & r < 1 \\ 0, & r > 1 \end{cases}$ $r = \sqrt{x^2 + y^2}$		$\frac{J_1(2\pi\rho)}{\pi\rho}$ $\rho = \sqrt{f_x^2 + f_y^2}$

