1. For dopant atoms uniformly distributed in a silicon crystal, how far apart are these dopant atoms when the doping concentration is a) $2 \times 10^{15} \text{ cm}^{-3}$, b) $10^{18} \text{ cm}^{-3}$, c) $7 \times 10^{20} \text{ cm}^{-3}$.

The average distance between dopant atoms is the cubed root of the concentration. Thus,

a) $d = (2 \times 10^{15} \text{ cm}^{-3})^{-1/3} = 0.794 \times 10^{-5} \text{ cm} \approx 80 \text{ nm}$ (keeping one digit of precision)

b) $d = (1 \times 10^{18} \text{ cm}^{-3})^{-1/3} = 1 \times 10^{-6} \text{ cm} = 10 \text{ nm}$

c) $d = (7 \times 10^{20} \text{ cm}^{-3})^{-1/3} = 1.13 \times 10^{-7} \text{ cm} \approx 1.1 \text{ nm}$

2. What is the resistivity of pure silicon at room temperature?

$$1 = \sigma = q(n\mu_n + p\mu_p), \quad n = p = n_i$$

$$\sigma = 1.6 \times 10^{-19} \mathcal{C} (1.5 \times 10^{10} \text{ cm}^{-3}) \left( 1500 \frac{\text{cm}^2}{V_s} + 450 \frac{\text{cm}^2}{V_s} \right) = 4.7 \times 10^{-6} (\Omega \text{ cm})^{-1}$$

$$\rho = 2.1 \times 10^5 \Omega \text{ cm}$$

(Note that silicon dioxide has a resistivity of about $10^{14} \Omega \text{ cm}$, and copper is about $10^{-6} \Omega \text{ cm}$.)

3. a) Show that the minimum conductivity of a semiconductor occurs when $n = n_i\sqrt{\mu_p/\mu_n}$.

Use the mass action equation $np = n_i^2$ to put $p$ in terms of $n$ in the conductivity equation.

$$\sigma = q \left( n\mu_n + \frac{n_i^2}{n} \mu_p \right)$$

Now take the derivative wrt $n$, set it equal to zero, and solve for $n$.

$$\frac{d\sigma}{dn} = q \left( \mu_n - \frac{n_i^2}{n^2} \mu_p \right) = 0, \quad n^2 = n_i^2 \frac{\mu_p}{\mu_n}, \quad n = n_i \sqrt{\frac{\mu_p}{\mu_n}}$$

3. b) How does the minimum conductivity for silicon compare to the intrinsic conductivity of silicon at room temperature?
Using the results of 3a in the equation for the conductivity, \( \sigma_{\text{min}} = 2 q n_i \sqrt{\mu_n \mu_p} \).

\[
\sigma = 2 (1.6 \times 10^{-19} \text{C})(1.5 \times 10^{10} \text{cm}^{-3}) \sqrt{\left(1500 \frac{\text{cm}^2}{\text{Vs}}\right) \left(450 \frac{\text{cm}^2}{\text{Vs}}\right)} = 3.9 \times 10^{-6} (\Omega \text{cm})^{-1}
\]

This is only a little smaller (16%) than the value for intrinsic silicon calculated in problem 2.

4. Consider a resistor made of pure silicon with a cross-sectional area of 0.5 \( \mu \text{m}^2 \), and a length of 50 \( \mu \text{m} \). What is the resistance of this silicon piece? For an applied voltage of 5 V, how much current would flow through it?

\[
R = \rho \frac{L}{A} = 2.1 \times 10^5 \Omega \text{cm} \frac{50 \mu \text{m}}{0.5 \mu \text{m}^2 \left(\frac{10,000 \mu \text{m}}{1 \text{cm}}\right)} = 2.1 \times 10^{11} \Omega
\]

(That’s a big resistance!)

\[
V = IR, \quad I = \frac{5V}{2.1 \times 10^{11} \Omega} = 24 \text{ pA}
\]

5. Suppose the resistor of problem 4 were made of p-type silicon. What doping level would be required to make the resistance equal to 25 k\( \Omega \)? 25\( \Omega \)?

The required conductivity is \( \sigma = \frac{1}{\rho} = \frac{1}{\frac{L}{RA}} = \frac{1}{25,000} \frac{50 \mu \text{m}}{0.5 \mu \text{m}^2} \left(\frac{10,000 \mu \text{m}}{1 \text{cm}}\right) = 40 (\Omega \text{cm})^{-1} \)

For p-type silicon, the electron concentration can be ignored and the conductivity will be

\[
\sigma = q \mu_p = q N_A \mu_p \quad \text{(since} \ p \approx N_A) \quad \text{Thus},
\]

\[
N_A = \frac{\sigma}{q \mu_p} = \frac{40 (\Omega \text{cm})^{-1}}{(1.6 \times 10^{-19} \text{C}) \left(\frac{450 \text{cm}^2}{\text{Vs}}\right)} = 5.6 \times 10^{17} \text{cm}^{-3}
\]

To make a 25\( \Omega \) resistor, the doping level would have to be 1000 time higher: \( 5.6 \times 10^{20} \text{cm}^{-3} \). This can be hard to do, since it is near the solid solubility limit for most dopants.