CHE323/384 Chemical Processes for Micro- and Nanofabrication
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Homework #10 Solutions

1. A photoresist gives a final resist thickness of 320 nm when spun at 2800 rpm.
   a) What spin speed should be used if a 290-nm-thick coating of this same resist is desired?
   b) If the maximum practical spin speed for 200-mm wafers is 4000 rpm, at what thickness would a lower viscosity formulation of the resist be required?

For resist thickness *d*, the impact of changing spin speed is given by $\frac{d_2}{d_1} = \frac{\sqrt{\omega_1}}{\sqrt{\omega_2}}$

a) The new spin speed needed would be $\omega_2 = \omega_1 \left(\frac{d_1}{d_2}\right)^2 = 2800\left(\frac{320}{290}\right)^2 = 3409\ rpm$.

b) The min thickness for the same viscosity would be $d_2 = d_1 \frac{\sqrt{\omega_1}}{\sqrt{\omega_2}} = 320\left(\frac{2800}{4000}\right)^{0.5} = 268\ nm$. For a thinner resist, a lower viscosity resist formulation would be required.

2. Complimentary mask features (for example, an isolated line and an isolated space of the same width) are defined by

$$t_m^c(x,y) = 1 - t_m(x,y)$$

Prove that the diffraction patterns of complimentary mask features are given by

$$T_m^c(f_x, f_y) = \delta(f_x, f_y) - T_m(f_x, f_y)$$

Use this expression to derive the diffraction pattern of an isolated line.

Taking the Fourier transform of the first equation:

$$\mathcal{F}\left\{t_m^c(x,y)\right\} = \mathcal{F}\{1\} - \mathcal{F}\{t_m(x,y)\} \Rightarrow T_m^c(f_x, f_y) = \delta(f_x, f_y) - T_m(f_x, f_y)$$

For a 1-D space, $T_m(f_x) = \frac{\sin(\pi w f_x)}{\pi f_x}$. Thus, the complimentary line has a diffraction pattern

$$T_m^c(f_x, f_y) = \delta(f_x, f_y) - \frac{\sin(\pi w f_x)}{\pi f_x}$$

3. Show that the Fourier transform is a linear operation, that is, show that for two functions *f*(*x*,*y*) and *g*(*x*,*y*), and two constants *a* and *b*,

$$\mathcal{F}\{af(x,y)+bg(x,y)\}=aF(f_x,f_y)+bG(f_x,f_y)$$

Applying the definition of the Fourier transform,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(af(x,y)+bg(x,y)\right)e^{-2\pi i(f_x x+f_y y)}dx\,dy$$
$$=a\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)e^{-2\pi i(f_x x+f_y y)}dx\,dy+b\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x,y)e^{-2\pi i(f_x x+f_y y)}dx\,dy$$
$$=aF(f_x,f_y)+bG(f_x,f_y)$$

4. Prove the shift theorem of the Fourier transform:

$$\text{If } \mathcal{F}\{g(x,y)\}=G(f_x,f_y),\quad \mathcal{F}\{g(x-a,y-b)\}=G(f_x,f_y)e^{-i2\pi(f_x a+f_y b)}$$

Applying the definition of the Fourier transform, then letting $x'=x-a$ and $y'=y-b$,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x-a,y-b)e^{-2\pi i(f_x x+f_y y)}dx\,dy=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x',y')e^{-2\pi i\left(f_x[x'+a]+f_y[y'+b]\right)}dx'\,dy'$$
$$=e^{-2\pi i\left(f_x a+f_y b\right)}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x',y')e^{-2\pi i\left(f_x x'+f_y y'\right)}dx'\,dy'=G(f_x,f_y)e^{-i2\pi(f_x a+f_y b)}$$

5. Prove the similarity theorem of the Fourier transform:

$$\text{If } \mathcal{F}\{g(x,y)\}=G(f_x,f_y),\quad \mathcal{F}\{g(ax,by)\}=\frac{1}{|ab|}G\left(\frac{f_x}{a},\frac{f_y}{b}\right)$$

Applying the definition of the Fourier transform, then letting $x'=ax$ and $y'=by$,

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(ax,by)e^{-2\pi i(f_x x+f_y y)}dx\,dy=\frac{1}{|ab|}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x',y')e^{-2\pi i\left(f_x[x'/a]+f_y[y'/b]\right)}dx'\,dy'$$
$$=\frac{1}{|ab|}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x',y')e^{-2\pi i\left(\frac{f_x}{a}x'+\frac{f_y}{b}y'\right)}dx'\,dy'$$

Why the absolute value sign out in front (the factor $1/|ab|$)? Suppose a is negative. Then the limits of the integration of x' would have to go from $+\infty$ to $-\infty$. Multiplying the integral by -1 would put the range of integration back to its original $-\infty$ to $+\infty$. Thus, the multiplier in front of the integral would be $-1/a$ if a is negative, and $+1/a$ if a is positive. This is equivalent to $1/|a|$. Of course, the same argument holds for b.