CHE323/384 Chemical Processes for Micro- and Nanofabrication
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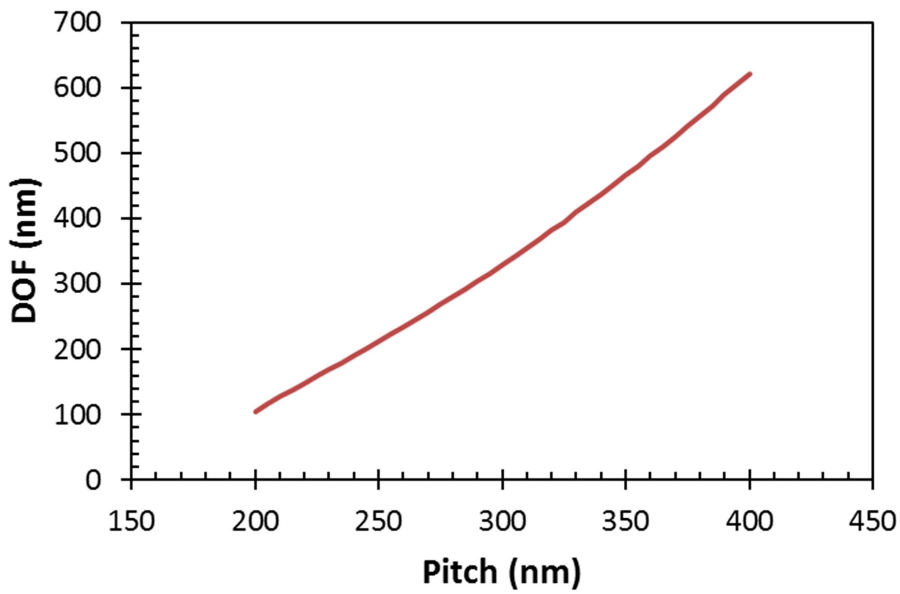
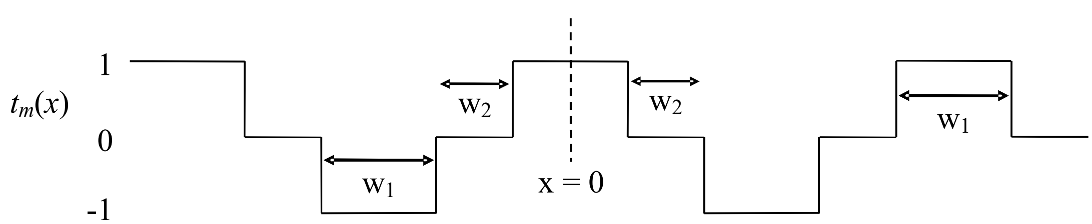
Homework #13 Solutions

1. Generate a plot of DOF versus feature size using the Rayleigh DOF criterion. Assume a 193 nm wavelength, equal lines and spaces, coherent three-beam imaging, and $k_2 = 0.8$.

$DOF = \frac{k_2}{2}\frac{\lambda}{n(1-\cos\theta)}$ (from lecture 48). Also, $\sin\theta = \frac{\lambda}{np}$ (position of first diffraction order).
Assuming imaging in air ($n = 1$), we can use a spreadsheet to vary p, giving different angles θ. Then using this angle in the DOF equation, we get DOF vs. pitch.



2. Derive the diffraction pattern of an alternating phase shift mask (a repeating pattern of lines and spaces where every other space is shifted in phase by 180°, resulting in a transmittance of −1):



The diffraction pattern for a repeating pattern of spaces of width w_1 and pitch $2p$, where $p = w_1+w_2$, centered at $x = 0$:

$$T_m(f_x) = \frac{1}{2p} \sum_{j=-\infty}^{\infty} \frac{\sin(\pi f_x w_1)}{\pi f_x} \delta\left(f_x - \frac{j}{2p} \right)$$

The diffraction pattern of a second repeating pattern of spaces of width w_1 and pitch $2p$, shifted by p and with transmittance -1:

$$T_m(f_x) = -\frac{1}{2p} \sum_{j=-\infty}^{\infty} \frac{\sin(\pi f_x w_1)}{\pi f_x} \delta\left(f_x - \frac{j}{2p} \right) e^{-i2\pi f_x p}$$

Using superposition to add these two arrays together:

$$T_m(f_x) = \frac{1}{2p} \sum_{j=-\infty}^{\infty} \frac{\sin(\pi f_x w_1)}{\pi f_x} \delta\left(f_x - \frac{j}{2p} \right)\left(1 - e^{-i2\pi f_x p}\right)$$

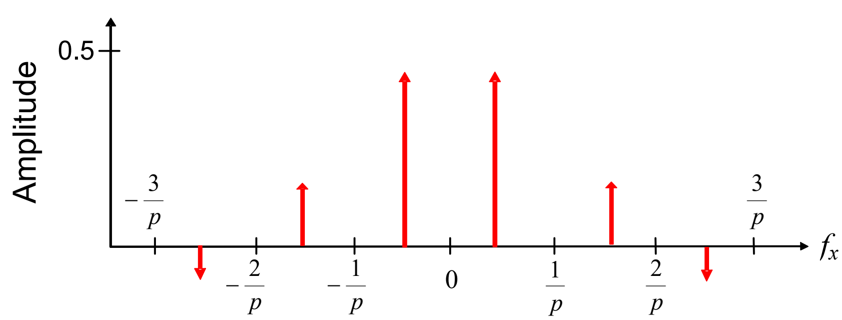
But, since $f_x = j/2p$ are the only non-zero values of the diffraction pattern, we can substitute this value everywhere for f_x:

$$T_m(f_x) = \sum_{j=-\infty}^{\infty} \left(1 - e^{-i(j\pi)}\right) \frac{\sin(j\pi w_1/2p)}{j\pi} \delta\left(f_x - \frac{j}{2p} \right) = \sum_{j=-\infty}^{\infty} \left(1 - (-1)^j\right) \frac{\sin(j\pi w_1/2p)}{j\pi} \delta\left(f_x - \frac{j}{2p} \right)$$

Notice that all the even values of j disappear.

To help illustrate this diffraction patter, let's graph the resulting diffraction pattern out to the $\pm 5^{th}$ diffraction orders for the case of $w_1 = w_2$. The diffraction pattern will have diffraction orders spaced out at intervals of $1/2p$ with amplitudes

$$a_j = \begin{cases} 0, & j = even \\ 2\frac{\sin(j\pi/4)}{j\pi}, & j = odd \end{cases}, \quad a_1 = a_{-1} = \frac{\sqrt{2}}{\pi}, \quad a_3 = a_{-3} = \frac{\sqrt{2}}{3\pi}, \quad a_5 = a_{-5} = -\frac{\sqrt{2}}{5\pi}, \quad \text{etc.}$$



3. Consider the coherent, in-focus 2-beam and 3-beam aerial images for equal lines and spaces:

2-beam: $$I(x) = \left[\frac{1}{2} + \frac{1}{\pi}\cos(2\pi x/p)\right]^2$$

3-beam: $$I(x) = \left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi x/p)\right]^2$$

Derive expressions for the NILS for each of these cases.

$NILS = w\dfrac{d\ln(I)}{dx}$. For both images, $I(x = p/4) = 1/4$.

For the 2-beam case, $\dfrac{dI}{dx} = 2\left[\frac{1}{2} + \frac{1}{\pi}\cos(2\pi x/p)\right]\left[-\frac{1}{\pi}\sin(2\pi x/p)\right](2\pi/p)$. At x = p/4, and ignoring the sign, $\dfrac{dI}{dx} = \dfrac{2}{p}$. Thus, $NILS = \dfrac{p}{2}\dfrac{1}{I}\dfrac{dI}{dx} = 4$.

For the 3-beam case, a similar derivations gives $NILS = 8$.

4. Consider a general expression for the aerial image of a line/space pattern.

$$I(x) = \sum_{j=0}^{N} \beta_j \cos(2\pi jx/p)$$

Derive an expression for the NILS for the case of equal lines and spaces.

The NILS calculated at the edge position $x = w/2 = p/4$ will be

$$NILS = -\pi\frac{\sum_{j=0}^{N} j\beta_j \sin(\pi j/2)}{\sum_{j=0}^{N} \beta_j \cos(\pi j/2)} = -\pi\frac{\beta_1 - \beta_3 + \beta_5 - ...}{\beta_0 - \beta_2 + \beta_4 - ...}$$

5. For the Mack model of development, what is the value of the development rate at $m = m_{th}$? For $m_{th} > 0$, what is the limit of this value as n becomes large?

$$r = r_{max}\frac{(a+1)(1-m)^n}{a+(1-m)^n} + r_{min}, \quad a = \frac{(n+1)}{(n-1)}\left(1-m_{th}\right)^n$$

At $m = m_{th}$, $r = r_{max} \dfrac{(a+1)(1-m_{th})^n}{a+(1-m_{th})^n} + r_{min} = r_{max} \dfrac{a+1}{\dfrac{a}{(1-m_{th})^n}+1} + r_{min}$

$$= r_{max} \frac{a+1}{\dfrac{(n+1)}{(n-1)}+1} + r_{min} = r_{max} \frac{(a+1)(n-1)}{2n} + r_{min} = r_{max} \frac{(n+1)(1-m_{th})^n + (n-1)}{2n} + r_{min}$$

As n gets very large, $(1-m_{th})^n$ goes to zero for $m_{th} > 0$. Thus, for this case, $r(m_{th}) \approx r_{max} \dfrac{n-1}{2n} + r_{min} \approx \dfrac{r_{max}}{2} + r_{min}$

6. Consider the case when the diffusion rate constant for the development mechanism is large compared to the surface reaction rate constant (i.e., the rate is reaction-controlled). If a » 1, show that the Mack development rate will become

$$r = r_{max}(1-m)^n + r_{min}$$

$$r = r_{max} \frac{(a+1)(1-m)^n}{a+(1-m)^n} + r_{min}, \quad a = \frac{(n+1)}{(n-1)}(1-m_{th})^n$$

For a » 1, $r = r_{max} \dfrac{a(1-m)^n}{a+(1-m)^n} + r_{min}$. Also, if a » 1 we must necessarily have $a >> (1-m)^n$ for all m. Thus, $r = r_{max} \dfrac{a(1-m)^n}{a} + r_{min} = r_{max}(1-m)^n + r_{min}$.