CHE323/CHE384
Chemical Processes for Micro- and Nanofabrication
www.lithoguru.com/scientist/CHE323

# Lecture 42
# Lithography: Diffraction, part 2

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**Reading**:
Chapter 7, *Fabrication Engineering at the Micro- and Nanoscale*, 4th edition, Campbell

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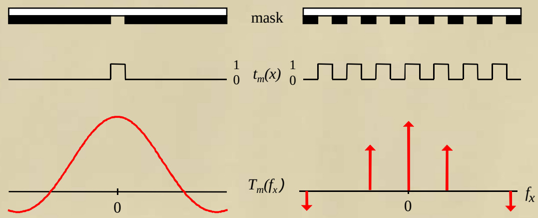
## Diffraction

Note: masks not drawn to scale.




**Isolated space**

$$T_m(f_x) = \mathcal{F}\{t_m(x)\} = \frac{\sin(\pi w f_x)}{\pi f_x}$$

**Equal lines and spaces**

$$T_m(f_x) = \frac{1}{p}\sum_{n=-\infty}^{\infty} \frac{\sin(\pi w f_x)}{\pi f_x}\,\delta\left(f_x - \frac{n}{p}\right)$$

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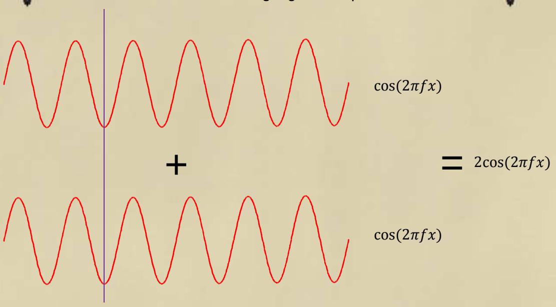
## Constructive Interference

Two waves adding together in phase



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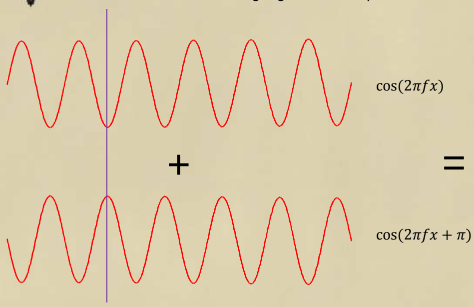
## Destructive Interference

Two waves adding together out of phase



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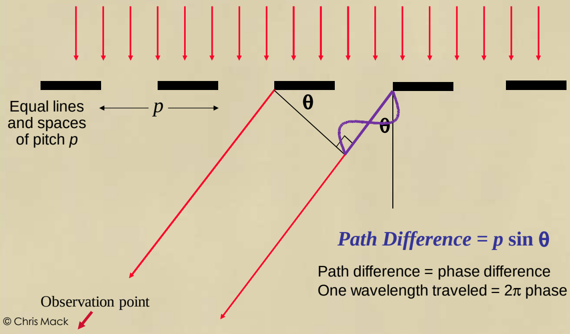
## Diffraction - example

**Coherent Illumination (normal incident plane wave)**




***Path Difference = p sin θ***

Path difference = phase difference
One wavelength traveled = 2π phase

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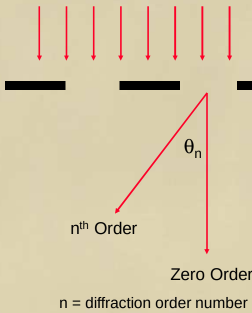
## Diffraction - example

**Coherent Illumination**




n = diffraction order number

Constructive interference occurs when the path difference is a multiple of the wavelength:

**Bragg's Condition:**

$$p\sin\theta_n = n\lambda$$

$$f = \frac{\sin\theta_n}{\lambda} = \frac{n}{p}$$

## Diffraction - example

- The magnitude of each diffraction order is calculated from Fraunhofer diffraction theory:

**For a repeating line/space pattern,**

$$a_n = \frac{\sin(n\pi w/p)}{n\pi}$$

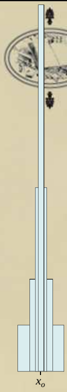
**where w = space width**
**p = pitch**

**For equal lines and spaces,**

$$T_m(f_x) = \sum_{n=-\infty}^{\infty} a_n \delta\left(f_x - \frac{n}{p}\right), \quad a_n = \frac{\sin(n\pi/2)}{n\pi} = \begin{cases} \frac{1}{2}, & n=0 \\ 0, & n=even \\ \frac{1}{n\pi}, & n=1,5,9,\ldots \\ -\frac{1}{n\pi}, & n=3,7,11,\ldots \end{cases}$$




## What's a Delta Function?




Consider a unit area rectangle centered at $x_o$.
Cut the width in half and double the height.
Repeat ad infinitum.

$$\delta_\varepsilon(x-x_o) = \begin{cases} 0, & x < x_o - \frac{\varepsilon}{2} \\ \frac{1}{\varepsilon}, & x_o - \frac{\varepsilon}{2} < x < x_o + \frac{\varepsilon}{2} \\ 0, & x > x_o + \frac{\varepsilon}{2} \end{cases} = \frac{1}{\varepsilon} rect\left[\frac{x-x_o}{\varepsilon}\right]$$

$$\delta(x-x_o) = \lim_{\varepsilon \to 0} \delta_\varepsilon(x-x_o)$$

Result: Function with zero width, infinite height, and unit area.
Used to represent an ideal point of light.




## Delta Function Sifting

- Defining properties of the delta function:

$$\delta(x) = 0 \quad when \quad x \neq 0$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

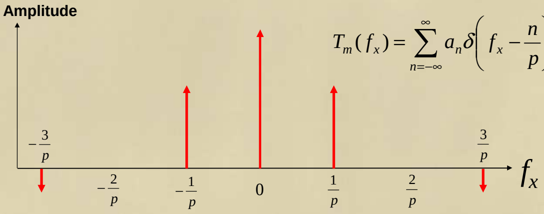
- The Sifting Property of the delta function:

$$\int_{-\infty}^{\infty} f(x)\delta(x-x_o)dx = f(x_o)$$




## Diffraction - example

- The diffraction pattern can be graphed as




$f_x = \sin\theta/\lambda$




## Diffraction

- Small features diffract more:

$$T_m(f_x) = \mathcal{F}\{t_m(x)\} = \frac{\sin(\pi w f_x)}{\pi f_x}$$




## Lecture 42: What have we Learned?

- Define constructive and destructive interference
- What is Bragg's Condition?
- What is a delta function, and what does it represent physically?
- What is a diffraction order and when do they show up in diffraction patterns?
- How does the size of a mask pattern affect its diffraction pattern?