Diffraction Review

- **Diffraction** is the propagation of light in the presence of boundaries
- In lithography, diffraction can be described by Fraunhofer diffraction - the diffraction pattern is the Fourier Transform of the mask transmittance function
- Small patterns diffract more: frequency components of the diffraction pattern are inversely proportional to dimensions on the mask
- For repeating patterns (such as a line/space array), the diffraction pattern becomes discrete diffracted orders
- Information about the pitch is contained in the positions of the diffracted orders, and the amplitude of the orders determines the duty cycle (w/p)

Fourier Transform Examples

<table>
<thead>
<tr>
<th>g(x)</th>
<th>Graph of g(x)</th>
<th>G(f)</th>
<th>Graph of G(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rect</td>
<td><img src="image1" alt="Graph of rect" /></td>
<td><img src="image2" alt="Graph of G(rect)" /></td>
<td><img src="image2" alt="Graph of G(rect)" /></td>
</tr>
<tr>
<td>step</td>
<td><img src="image3" alt="Graph of step" /></td>
<td><img src="image4" alt="Graph of G(step)" /></td>
<td><img src="image4" alt="Graph of G(step)" /></td>
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<tr>
<td>δ(x)</td>
<td><img src="image5" alt="Graph of δ(x)" /></td>
<td>![Graph of G(δ(x))] (image6)</td>
<td>![Graph of G(δ(x))] (image6)</td>
</tr>
<tr>
<td>comb</td>
<td><img src="image7" alt="Graph of comb" /></td>
<td><img src="image8" alt="Graph of G(comb)" /></td>
<td><img src="image8" alt="Graph of G(comb)" /></td>
</tr>
</tbody>
</table>

Fourier Transform Properties

- **Linearity:** \( \mathcal{F}\{af(x,y) + bg(x,y)\} = a\mathcal{F}\{f(x,y)\} + b\mathcal{F}\{g(x,y)\} \)
- **Shift Theorem:** 
  \[
  \mathcal{F}\{e^{j2\pi \frac{b}{a} x} f(x,y)\} = \mathcal{F}\{f(\xi,\eta)\} e^{-j2\pi \frac{b}{a} \xi} 
  \]
- **Similarity:** 
  \[
  \mathcal{F}\{f(ax,by)\} = \frac{1}{|ab|} \mathcal{F}\{f(x,y)\} 
  \]
- **Convolution:** 
  \[
  \mathcal{F}\{G(f(x,y)) = \mathcal{F}\{f(x,y)\} \mathcal{F}\{g(x,y)\} 
  \]

**Numerical Aperture**

\[ NA = n \sin \alpha \]

- \( n \) is index of refraction of media (air in this case)
- \( \alpha \) is maximum half-angle of light making it through the lens

**F - Number**

\[
F/# = \frac{b}{a} = \frac{1}{2NA} 
\]

- The F/\# or F-Stop is used by cameras as an alternative to the Numerical Aperture
**Magnification/Reduction**

- Given the reduction ratio $R$:
  \[
  n_m \sin \theta_m = R n_m \sin \theta_m
  \]

**Entrance Pupil** - the image of the aperture stop as seen from the front of the lens

**Exit Pupil** - the image of the aperture stop as seen from the back of the lens

**Numerical Aperture**

- Graphing the diffraction pattern with the aperture:

**Objective Lens Aperture**

Top down view:

- $0^{th}$ Order
- $+1^{st}$ Order
- $-1^{st}$ Order
- $0^{th}$ Order
- $+1^{st}$ Order
- $-1^{st}$ Order

**Forming an Image**

- The objective lens focuses the diffraction orders onto the wafer

- The image plane is defined as the plane where all diffraction orders arrive "in phase"

**Fourier Optics**

- The objective lens produces an image equal to the Fourier transform of the light entering the lens.

- The combination of diffraction followed by the imaging lens produces a Fourier transform of a Fourier transform, which gives back the original pattern.

- However, the objective lens cuts off the high spatial frequencies of the diffraction pattern.
Fourier Optics

• The objective lens is described by its pupil function, \( P(f_x, f_y) \). For an ideal lens, this function just describes the size of the aperture.

\[
P(f_x, f_y) = \begin{cases} 
1, & \text{when } \sqrt{f_x^2 + f_y^2} \leq \text{NA} \lambda \\
0, & \text{otherwise}
\end{cases}
\]

• Given a diffraction pattern \( T_m \) and a pupil function \( P \), the light which makes it into the lens is just \( PT_m \).

• The lens then takes the inverse Fourier transform of this light to give the electric field of the image, \( E \).

\[
E(x, y) = F^{-1}\{PT_m\} \\
I(x, y) = |E(x, y)|^2
\]

Lecture 43: What have we Learned?

• Define “numerical aperture”
• What are the entrance and exit pupils of an imaging lens?
• How can one determine which diffraction orders pass through an imaging lens?
• What is the pupil function of a lens?
• Explain the concept of Fourier Optics