Response Surface Modeling

Beyond Factorial Designs

- A two-level factorial (or fractional factorial) design looks for linear trends, possibly with interactions
  - Does the variable significantly impact the response?
  - In what direction?
  - It helps to define the next experiment
- Response Surface Methodology (RSM) looks for quadratic or higher order trends
  - Assumes all variables are significant
  - A quadratic response always has a stationary point (minimum or maximum or saddle point)
  - Can be used to optimize a process

General Second Order Model

\[ y_i = \beta_0 + \sum_{j=1}^{k} \beta_j x_{ij} + \sum_{j=1}^{k} \sum_{j \neq j}^{k} \beta_{ij} x_{ij} x_{ij} + \epsilon_i \]

In matrix form:

\[ Y = \beta_0 + Xb + XB\beta^T + \epsilon \]

One at a Time Variables

- What are the optimum process conditions?

Central Composite Design

- Take the two-level factorial design and add:
  - A center point (the middle point between all of the factors), usually with repeats
  - Axial (star) points (the center point except with one variable changed to be at +/- an extreme value). Do this for all variables.
  - More efficient than a three-level factorial design
Central Composite Designs

- Central Composite Circumscribed
  \[ \text{This distance} = 2^{k/4} \]
- Central Composite Face-centered
  \[ \text{For } k \text{ factors, } n = 2^k + 2k + 1 \]

Box-Behnken Design

- Put a data point in the center, then one data point at the midpoints of each edge of the process space
  - Does not contain an embedded factorial design
  - No corner (extreme) points

Repeated Center Points

- Repeated center points are not randomized; they are run as the first and last data points, and evenly spread through the rest of data collection
  - A check against process instability
  - All other points should have randomized order
- The number of repeated center points can be set to create “uniform precision”
  - The variance of prediction is the same at the center as it is at the corners

RSM Properties

- Orthogonality: Are the factors correlated?
- Rotatability: A design is rotatable if the variance of the predicted response at any point \(x\) depends only on the distance of \(x\) from the design center point
  - All first-order orthogonal designs are rotatable
  - Composite face-centered design is not rotatable
- Uniformity: control the number of center points to achieve uniform precision
- Efficiency: how many experimental runs are required?

Notes on RSM

- Beware of extrapolation: is the stationary point (min or max) outside of the experimental space?
- Multiple responses: try overlapping the response contour plots
- Principle component analysis can be useful with RSM

Lecture 71: What have we learned?

- When is response surface modeling preferred over two-level factorial design?
- What is wrong with varying our factors one-at-a-time?
- Describe some of the RSM designs we have discussed
- Why are center points often repeated in RSM?
- What are the four properties we look for in RSM?