All Measurements are Uncertain

- Measurement error exists, but we do not know what it is
  - If we knew the measurement error, we would subtract it out!
  - Unknown errors are called uncertainties
- Our goal is to estimate the uncertainty in our measurements
  - Random errors can be estimated using repeated measurements
  - Systematic errors require a sophisticated understanding of the measurement process

Why Evaluate Uncertainty?

- To allow realistic comparison of results
  - Different experiments, experimenters, over time, etc.
- To allow for more rigorous use of the data
  - Regression, model building, etc.
- Understanding the sources of uncertainty allows one to validate, and possibly improve, measurement procedures
  - How can we reduce uncertainty?
- Evaluating uncertainty means you can report uncertainty — a requirement of good science

What is Data?

Data = the results of a measurement

- Definition of the thing being measured
- Measurement value (number plus units)
- Estimate of the uncertainty of each measurement
- Experimental context (measurement method + environment)
- Context uncertainty (uncertainty of controlled and uncontrolled input parameters)
- Measurement model (theory, assumptions and definitions used in making the measurement)
Uncertainty Components

- **Systematic errors**
  - Produce a bias in the measurement result
  - We look for and try to correct for all systematic errors, but we are never totally successful
  - We try to put an upper limit on how big we expect systematic errors could be
- **Random errors**
  - Can be evaluated statistically, through repeated measurements

**Other Measurement Terms**

- **Accuracy**: the same as error, it can never be known, but is estimated as the maximum systematic error that might exist
- **Precision**: the standard deviation of repeated measurements (random error component)
- **Repeatability**: standard deviation of repeated measurements under conditions as nearly identical as possible
- **Reproducibility**: standard deviation of repeated measurements under conditions that vary in a typical way (different operators, instruments, days)

**Uncertainty Types**

- **Type A**: uncertainty estimates obtained by statistical analysis of repeated measurements
  - Standard uncertainty = standard deviation of repeated measurements
  - Fundamentally caused by fluctuations in nature (shot noise, Brownian motion, Boltzmann statistics, etc.) that propagate through the measurement model
- **Type B**: uncertainty estimates evaluated by other techniques (scientific judgment)
  - Prior experience or data, manufacturer’s specs, calibration reports, reported uncertainty value

**Sources of Type B Uncertainty**

- Incomplete definition of the measurement
- Imperfect realization of the procedure
- Sample is not representative
- Environmental conditions
- Biases in reading analog scales
- Instrument resolution
- Values of constants used in calculations
- Changes in measuring instrument performance since last calibration
- Approximations/assumptions in the measurement model

**Combined Uncertainty \((u_c)\)**

- Combine individual standard uncertainties \((u_i)\) arising from Type A and/or Type B evaluation (including their covariances)

\[
u_c^2 = u_1^2 + u_2^2 + 2\text{cov}(1,2)
\]

- We usually use a propagation of uncertainties approach

**Expanded Uncertainty**

- The combined standard uncertainty \((u_c)\) is a standard deviation
- We often express our uncertainty as a multiple of the standard deviation

\[
U = ku_c
\]

- \(U\) = expanded uncertainty
- \(k\) = coverage factor (typically 2, or up to 3)
Effective Degrees of Freedom

- When using the combined uncertainty, we usually assume that our measurement is t-distributed.
- The effective number of degrees of freedom is given by the Welch-Satterthwaite approximation:

\[
d_{eff} = \frac{\left(\sum u_i^2\right)^2}{\sum (u_i^4 / d_i)}
\]

- We often use this to calculate a coverage factor for a given probability (e.g., 95%).